**Option Greeks**

**Introduction**

# What are Greeks?

* They measure the **sensitivity of the price (value)** of a **European option to a change in a factor**
* Measured through **Partial Derivatives** of the **Black Scholes Formula**

|  |  |  |
| --- | --- | --- |
| **Greek** | **Factor** | **Partial Derivative** |
|  | Price of the underlying |  |
|  | **Change in Delta** per unit change of the price of the underlying |  |
|  | Time to expiration |  |
|  | Volatility of the underlying |  |
|  | Risk Free Rate |  |
|  | Dividend yield of the underlying |  |

## 

* Measures the sensitivity of the option price to a **change in the price of the underlying**
* 
  + 
    - As **stock prices increase**, the value of calls **increase** thus their price increases (**Positive relationship**)
  + 
    - As **stock prices increase**, the value of puts **decrease** thus their price decreases (**Negative relationship**)

### Using Black Scholes Equation

|  |  |
| --- | --- |
| **Calls** | **Puts** |
|  |  |
|  |  |
|  |  |

We can relate the two using **Put Call Parity**,





## 

* Measures the sensitivity of the option Delta to a **change in the price of the underlying**
* It is the only Greek **not directly related** to the price of the option – thus it can be expressed as a **second derivative of the option price**
* 

For **both** Calls & Puts,



Note that this is a result of differentiation,



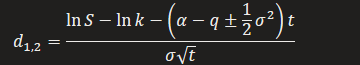


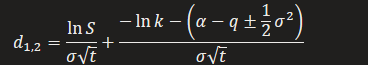














We can verify this relationship using Put Call Parity,









## 

* Measures the sensitivity of the option price to a **change in the time to expiration**
* Theta is **generally negative** – as time to expiration decreases, there is **less opportunity** for the option to move in the money thus becoming **less valuable**
* 

For Calls **specifically**,



Using **Put Call Parity**, we can derive the same for Puts,







* Measures the sensitivity of the option price to a **change in the volatility of the underlying**
* 

For **both** Calls & Puts,



We can verify this relationship using **Put Call Parity**,







## 

* Measures the sensitivity of the option price to a **change in the risk-free rate**
* 

For Calls specifically,



Using **Put Call Parity**,





Thus, we derive the formula for **Puts**,









## 

* Measures the sensitivity of the option price to a **change in the dividend yield of the underlying**
* 

For Calls **specifically**,



Using **Put Call Parity**,





Thus, we derive the formula for puts,









|  |  |  |  |
| --- | --- | --- | --- |
| **Greek** | **Long Call** | **Magnitude** | **Long Put** |
|  | + | > | - |
|  | + | = | + |
|  | - | < | - |
|  | + | = | + |
|  | + | > | - |
|  | - | < | + |

Since Long and Short Positions are opposite - their **Greeks must be opposite as well**



## Portfolio Level Greeks

* The Greeks we have discussed so far were for individual options
* To calculate the Greeks of a portfolio, we multiply each Greek by the **NUMBER** of options held and that option’s Greek



# Approximating Changes

* Fundamentally, Greeks measure the change in the Value of the Option with respect to a change in a variable
* If we know the change in the underlying variable and the Greek, we can approximate the change in the Option Value





Note that this can occur at a **portfolio level as well**, where we use the Portfolio Greek and Value of the entire portfolio of Options.

# Delta-Gamma-Theta Approximation

* Suppose we want to *estimate* the change in option price given a change in certain variables
* 
  + 
  + 
  + This is only true for small changes in the Underlying's Price
* 
  + All the other Greeks can be added as well, but the idea is for a **simple approximation**
  + Adding too many Greeks would complicate it, which in that case we might as well recalculate the actual option price
  + Similar concept to Duration Approximation





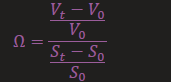
Similarly, we can also obtain the profit,



**Elasticity & Other Metrics**

# 

* Refers to the **percentage change** in the Option Price per percentage change in the Underlying Price
* Note that it can be related to the Delta of the option







|  |  |
| --- | --- |
| **Calls** | **Puts** |
|  |  |

# Portfolio Elasticity

* Refers to the percentage change of the Portfolio's Value per percentage change in the Underlying Price

|  |  |
| --- | --- |
| **Any Asset** | **Different Asset** |
|  |  |
| Faster; Preferred | Long Winded |

# Other Option Metrics

## Return (Expectation)







## Risk (Variance)



## Risk Premium







## Sharpe Ratio







**Hedging**

# Delta Hedging

* Market Makers earn off Commissions and Bid Ask Spreads - they DO NOT want any capital appreciation or loss
* Thus, they can neutralize their risk by **Delta Hedging** their portfolio of Options
* Achieved by **buying/selling the underlying** to make the **Delta of the portfolio 0** - regardless of the change in the price of the underlying, the portfolio should remain neutral













* Unlike most other formulas, this uses NUMBER rather than weight
* 

calls 
Short the underlying 
Long the underlying 

## Initial Cashflow

* Market Makers would like to have a **zero initial cashflow** for the hedged portfolio
* Thus, they simply **Long or Short a Zero Coupon bond** at time 0 to offset the initial cashflow
* Since the Delta of a Bond is zero (Fixed Cashflow), the addition of a Bond does not affect the overall hedge of the portfolio

## Profit & Loss

* Using first principles, the profit of the portfolio is simply the **aggregate profit** of the overall position if the **position were closed**
  + Profit on Option Position
  + Profit on Underlying Position
  + Profit on Zero Coupon Bond
* There is **no need to memorize** any specific formula for this part as it is highly intuitive - but there is a formula involving the Delta Gamma Theta approximation for the Profit on the Option Position
* Despite the hedge, there is still a profit or loss on the position, reflecting the idea that Delta is **NOT a perfect estimator** of the option price (Recall Delta Gamma Theta Approximation)

## Break Even Point

* If the stock price changes by **exactly one standard deviation**, the profit will be 0
* The formal proof for this is too complicated; it is sufficient to know the result

|  |  |
| --- | --- |
| **Binomial Tree Approach** | **Black Scholes Framework** |
|  |  |

Overnight Profit ($) 
s 
one one 
s.d. s.d. 
Stock Price ($) 

# Rebalancing Hedge

* Delta changes whenever the price of the underlying changes, thus there is a **need to re-hedge** the portfolio if the price of the underlying changes









# Hedging Other/Multiple Greeks

* Other than just Delta, it is possible to hedge the portfolio based on additional greeks as well
* We implicitly assume that stock prices are NOT a function of anything else - any other Greek of a stock will be 0
* Thus, we CANNOT use the underlying to hedge for other Greeks - we need to use OTHER options to hedge the portfolio



To hedge the portfolio based on the two greeks,











Solve the **system of equations** to obtain the number of underlying and options to fully hedge the portfolio.